Exercise 7.1

Wiedemann-Franz law. (a) The electrical resistivity of gold at 300 K is $3.107 \times 10^{-8}\,\Omega\text{m}$, estimate its thermal conductivity at the same temperature; (b) The thermal conductivity of copper is $401\,\text{Wm}^{-1}\text{K}^{-1}$ at 300 K, estimate its electrical conductivity at the same temperature.

Solution

Wiedemann-Franz law establishes the relationship between electrical conductivity and electronic contributions to thermal conductivity, written as

$$L = \frac{\kappa_e}{\sigma T}.$$

L is the so-called called Lorentz number and has the value of $2.45 \times 10^{-8}\,\mathrm{W\Omega K^{-2}}$.

(a) Since the inverse of electrical conductivity is its resistivity, the thermal conductivity of gold is thus

$$\kappa_e = L\sigma T = \frac{2.45 \times 10^{-8} \times 300}{3.107 \times 10^{-8}} \approx 236.56\,\mathrm{Wm^{-1}K^{-1}}.$$

(b) The electrical conductivity of copper is thus

$$\sigma = \frac{\kappa_e}{LT} = \frac{401}{2.45 \times 10^{-8} \times 300} \approx 5.46 \times 10^{7}\,\mathrm{Sm^{-1}}.$$

Exercise 7.2

Energy and momentum relaxation time. The electrical resistivity and thermal conductivity of gold at 300K are $3.107 \times 10^{-8} \Omega \cdot m$ and $315 W m^{-1} K^{-1}$. Estimate the momentum and energy relaxation time, and the momentum and energy relaxation length of electrons in gold.

Solution

Following the Drude formula, the electrical conductivity can be written as,

$$\sigma = \frac{ne^2}{m_e}\tau$$

where n, m_e, and τ are electron density, electron mass and electron momentum relaxation time. Considering free electron mass and electron density($5.9 \times 10^{28}/m^3$) of gold at 300K, the momentum relaxation time of electron is

$$\tau = \frac{1}{\rho}\frac{m_e}{ne^2} \approx 19fs$$

As only the electrons at the Fermi surface contributes the conductance, the momentum relaxation length of electron can be calculated considering the Fermi velocity the electron at the Fermi level (Fermi energy(E_F) of gold corresponds to $5.53eV$).

$$l = \tau\nu_F = \tau \cdot (\frac{m_e}{2E_F})^{\frac{1}{2}} \approx 27nm$$

Energy relaxation time (τ_E) of electron in Au can be derived from electronic contribution in Au thermal conductivity,

$$k_e = \frac{1}{3}C_e\nu_F^2\tau_E$$

Considering the specific heat of electron ($\frac{1}{2}\pi^2 nk_BT/T_F$) at 300K, energy relaxation time of electron can be calculated to be,

$$\tau_E = \frac{3m_e k_e}{\pi^2 nk_B^2 T} \approx 2.59fs$$

Similarly, the energy relxation length can be computed as,

$$l_E = \tau_E\nu_F = \tau_E \cdot (\frac{m_e}{2E_F})^{\frac{1}{2}} \approx 36nm$$

Exercise 7.3

Electrons in semiconductors. An n-type semiconductor has a carrier concentration of $10^{18} cm^{-3}$ and a mobility of $200 cm^2 V^{-1} s^{-1}$ at 300K. Estimate the following:(a) electrical conductivity;(b) electron diffusivity;(c)momentum relaxation time; and (d)electron mean free path. Take the electron effective mass as that of a free electron.

Solution

(a) Electrical conductivity : Following the Drude formula of electrical conductivity, we get,

$$\sigma = n \cdot e \cdot \mu = 10^{18}[/cm^3] \cdot 1.6 \times 10^{-19}[C] \cdot 200[cm^2/Vs] = 0.32[S/m]$$

(b)Electron diffusivity : With the Einstein relation for the particle diffusion,

$$D_e = \frac{\mu_e k_B T}{e} = \frac{200[cm^2/Vs] \cdot 8.617 \times 10^{-5}[eV/K] \cdot 300K}{e} = 5.17[cm^2/Vs]$$

(c) Momentum relaxation time : With the Drude formula, charged particle mobility is proportional to the relaxation time.

$$\tau = \frac{m_e \mu_e}{e} = \frac{9.101 \times 10^{-31}[kg] \cdot 200[cm^2/Vs]}{1.602 \times 10^{-19}[C]} \approx 0.113ps$$

(d)Electron mean free path : In the absence of electric field in the material, electrons in the semiconductor will move randomly with thermal velocity($\nu_{th} = (\frac{3k_BT}{m})^{1/2}$). The mean free path of electron in this case is,

$$l = \tau \nu_{th} \approx 13.2[nm]$$